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Tensor Sum Operator and Dynamical System in Measurable Function Spaces

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Abstract- Let X be a Hausdroff topological space and let $B(E)$ be the Banach algebra of all bounded linear operators on a Banach space E. Let V be a system of weights on X. In this paper, tensor sum operator induced dynamical system in Measurable function space is introduced with applicable examples.

Key Words- System of weights, measurable function, weighted tensor sum operators, composition operators, dynamical system. 2000 AMS subject classification : 47B 37, 47 B 38, 47 B 07, 47 D 03, 30 H 05.

I. Introduction

Let X be a Hausdorff topological space and $M(X,E)$ be the space of all measurable function from X into E and $C(X,E)$ be the vector space of $M(X,E)$ consisting of the continuous function f from X into E. Let V be a set of non-negative upper- semi continuous functions on X. If V is a set of weights on X such that given any $x \in X$, there is some $v \in V$ for which $v(x) > 0$. We write$V > 0$.

A set V of weights on X is said to be directed upward provided for every pair u_1, u_2 in V and $\alpha > 0$ there exists $v \in V$ such that $\alpha u_i \leq v$ (point-wise on X) for $i = 1,2$.

By a system of weights, we mean a set V of weights on X with additionally satisfies $V > 0$. Let $cs(E)$ be the set of all continuous functions from X into E.,

If V is a system of weights on X then the pair(X,V) is called the weighted topological system. Associated with each weighted topological system (X,V), we have the weighted spaces of continuous E −valued functions defined as:

$$MV_0(X,E) = \{f \boxplus g \in M(X,E) \boxplus M(X,E): vq(f \boxplus g) \text{vanishes at } g \text{ on } X \text{ for each } v \in V, q \in cs(E)\}$$

$$MV_p(X,E) = \{f \boxplus g \in M(X,E) \boxplus M(X,E): vq(f \boxplus g) \text{ in } L^p \text{ for all } v \in V, q \in cs(E)\}$$

$$MV_b(X,E) = \{f \boxplus g \in M(X,E) \boxplus M(X,E): vq\big((f \boxplus g)(x)\big) \text{ is bounded in E for all } v \in V, q \in cs(E)\}$$

Let $v \in$ V, q $\in$ cs(E) and $f \boxplus g \in M(X,E) \boxplus M(X,E)$. If we define $\|f \boxplus g\|_{v,q} = \sup \{(\int_X (v(x)q(f \boxplus g)(x))^p d\mu)^{\frac{1}{p}}$ for all $x \in X\}$, then $\|.\|_v$ can be regared as a semi norm on either $MV_0(X,E) \boxplus MV_0(X,E), MV_b(X,E) \boxplus MV_b(X,E)$ and the family $\{\|.\|_{v,q}: v \in V, q \in cs(E)\}$ of semi norms defines a Hausdorff locally convex topology on each of these spaces. This topology will be dented by w_v and the vector spaces $MV_0(X,E)$ and $MV_b(X,E)$ endowed with w_v are called the weighted locally convex space of vector-valued continuous functions. It has a basis of closed absolutely convex neighborhoods of the origin of the form,

$$B_{v,q} = \{f \boxplus g \in MV_b(X,E) \boxplus MV_b(X,E): \|f \boxplus g\|_{v,q} \leq 1\}$$

Also, $MV_0(X,E)$ is a closed subspace of $MV_b(X,E)$.

1.1.Functions inducing tensor sum operators on weighted spaces of measurable functions.

In this section, let us investigate the Functions inducing tensor sum operators on weighted spaces of measurable functions.

Theorem:1.1.1. Let $\varphi: X \to X$ and $\pi_t : X \to \mathbb{C}$ be a measurable functions. Then $(C_\varphi \boxplus M_{\pi_t})(f \boxplus g)$ is a tensor sum operator for every $t \in \mathbb{R}$, $f \boxplus g \in MV_0(X) \boxplus MV_0(X)$ iff $V|C_\varphi \boxplus M_{\pi_t}| \leq V$.

Proof: First suppose $V|C_\varphi \boxplus M_{\pi_t}| \leq V$. Then For every $v \in V$, Then for all $v \in V$, there exist $u \in V$ such that $v|C_\varphi \boxplus M_{\pi_t}| \leq u$ (point wise on X). We show that $C_\varphi \boxplus M_{\pi_t}$ is a continuous linear operator on $MV_0(X) \boxplus MV_0(X)$. Clearly, $C_\varphi \boxplus M_{\pi_t}$ is linear on $MV_0(X) \boxplus MV_0(X)$. In order to prove the continuity of $C_\varphi \boxplus M_{\pi_t}$ on $MV_0(X) \boxplus MV_0(X)$, it is enough to show that $C_\varphi \boxplus M_{\pi_t}$ is continuous at origin. For this, suppose $f_\alpha \boxplus g_\alpha$ be a net in $MV_0(X) \boxplus MV_0(X)$ such that $P_v(f_\alpha \boxplus g_\alpha) \to 0$, for every $v \in V$.

Now,

$$P_v(C_\varphi f_\alpha \boxplus M_{\pi_t} g_\alpha) = P_v(C_\varphi f_\alpha) \boxplus P_v(M_{\pi_t} g_\alpha) \text{ for all } t \in \mathbb{R}.$$

$$= P_v(f_\alpha \circ \varphi(x)) \boxplus P_v(\pi_t(x) g_\alpha(x))$$

$$= (\int_X (v(x)q(f_\alpha(\varphi(x)))^p d\mu)^{\frac{1}{p}} \boxplus (\int_X (v(x)q(e^{th(x)} g_\alpha(x)))^p d\mu)^{\frac{1}{p}}$$

$$= (\int_X (v(x)q(f_\alpha(x)))^p d\mu)^{\frac{1}{p}} \boxplus (\int_X (v(x)q(e^{th(x)} g_\alpha(x)))^p d\mu)^{\frac{1}{p}}$$

$$= (\int_X (u(x)q(f_\alpha(x)))^p d\mu)^{\frac{1}{p}} \boxplus (\int_X \left(u(x)q(g_\alpha(x))\right)^p d\mu)^{\frac{1}{p}}$$

$$as\ t \to 0$$

$$= P_u(f_\alpha \boxplus g_\alpha) \to 0$$

This proves the continuity of $C_\varphi \boxplus M_{\pi_t}$ at origin and hence $C_\varphi \boxplus M_{\pi_t}$ is continuous on $MV_0(X) \boxplus MV_0(X)$.

Conversely, suppose $C_\varphi \boxplus M_{\pi_t}$ is continuous linear operator on $MV_0(X) \boxplus MV_0(X)$.We shall show that $V|C_\varphi \boxplus M_{\pi_t}| \leq V$. Let $v \in V$. Since $C_\varphi \boxplus M_{\pi t}$ is continuous origin, there exist $u \in V$ such that $(C_\varphi \boxplus M_{\pi_t})(B_u) \subseteq B_v$. We claim that $v|C_\varphi \boxplus M_{\pi_t}| \leq 2u$. Take $x_0 \in X$ and set $u(x_0) = \varepsilon$. In case $\varepsilon > 0$, $N = \{x \in X: u(x) < 2\varepsilon\}$ is an open neighborhood of x_0. Then there exists $f \boxplus g \in MV_0(X) \boxplus MV_0(X)$ such that $0 \leq f \boxplus g \leq 1$, and $f \boxplus g(X - N) = 0$.

Let $h = (2\varepsilon)^{-1}(f \boxplus g)$. Then clearly $h \in B_u$. Since $(C_\varphi \boxplus M_{\pi_t})(B_u) \subseteq B_v$, we have $(C_\varphi \boxplus M_{\pi_t})h \in B_v$ and this yields that $v(x)|(C_\varphi \boxplus M_{\pi_t})(x)||h(x)| \leq 1$, for all $x \in X$. From this it follows that $v(x)|(C_\varphi \boxplus M_{\pi_t})(x)||f \boxplus g(x)| \leq 2\varepsilon$, for all $\in X$.

Now suppose $u(x_0) = 0$ and that $v(x_0)|(C_\varphi \boxplus M_{\pi_t})(x_0)| > 0$ If we put that $\varepsilon = v(x_0)|(C_\varphi \boxplus M_{\pi_t})(x_0)|$ it is not greater than two and set, $N = \{x \in X: u(x) < \varepsilon\}$ then N would be an open neighborhood of x_0 and we could again find $f \boxplus g \in MV_0(X) \boxplus MV_0(X)$ such that $0 \leq f \boxplus g \leq 1$, and $f \boxplus g(x_0) = 1$ and $f \boxplus g(X - N) = 0$.Now let $h = \varepsilon^{-1}(f \boxplus g)$. Then clearly $h \in B_u$ and $(C_\varphi \boxplus M_{\pi_t})(h) \in B_v$. Hence $v(x)|(C_\varphi \boxplus M_{\pi_t})(x)||h(x)| \leq 1$ for all $x \in X$. This implies that $v(x)|(C_\varphi \boxplus M_{\pi_t})(x)||f \boxplus g(x)| \leq \varepsilon$, for all $x \in X$. From this it follows that, $v(x_0)|(C_\varphi \boxplus M_{\pi_t})(x_0)| \leq \frac{v(x_0)|(C_\varphi \boxplus M_{\pi_t})(x_0)|}{2}$ which is impossible. This proves our claim and hence the proof is complete.

Now we shall characterize tensor sum operators on $MV_0(X,E) \boxplus MV_0(X,E)$ induced by scalar-valued and vector valued functions.

1.2 Characterization of tensor sum operator

In this section, let us investigate the Characterization of tensor sum operator

Theorem: 1.2.1. Let $: X \to X$ and $\pi_t : X \to \mathbb{C}$ be a measurable function. Then $(C_\varphi \boxplus M_{\pi_t})(f \boxplus g)$ is a tensor sum operator for every $t \in \mathbb{R}$, $f \boxplus g \in MV_0(X,E) \boxplus MV_0(X,E)$ iff $V\|C_\varphi \boxplus M_{\pi_t}\| \leq V$.

Proof: Similar to proof of theorem 1.1.1.

Theorem:1.2.2. Let E be a (locally multiplicative convex) lmc algebra with unit e and let $\varphi : X \to E$ and $\pi_t : X \to \mathbb{C}$ be a bounded measurable function. Then $(C_\varphi \boxplus M_{\pi_t})(f \boxplus g)$ is a tensor sum operator on $t \in \mathbb{R}$, $f \boxplus g \in MV_0(X,E) \boxplus MV_0(X,E)$ iff $V_p \circ (\varphi \boxplus \pi_t) \leq V \; for \; all \; p \in P$.

Proof: Suppose $V_p \circ (\varphi \boxplus \pi_t) \leq V$, for all $p \in P$. Then for all $v \in V$, there exists $u \in V$ such that $v_p \circ (\varphi \boxplus \pi_t) \leq u$ (point wise on X). We shall prove that the mapping $\varphi : X \to E$ and $\pi_t : X \to \mathbb{C}$ gives rise to a linear transformation $C_\varphi \boxplus M_{\pi_t}$ from $MV_0(X,E) \boxplus MV_0(X,E)$ itself defined as $C_\varphi f = \varphi f \; and \; M_{\pi_t} g = \pi_t g$ for every $f \boxplus g \in MV_0(X,E) \boxplus MV_0(X,E)$, where the product is point-wise continuous linear operator on $MV_0(X,E)$, we shall establish the continuity of $C_\varphi \boxplus M_{\pi_t}$ at the origin. For this, let $\{f_\alpha \boxplus g_\alpha\}$ be a net in $MV_0(X,E) \boxplus MV_0(X,E)$ such that for all $v \in V, p \in P$, $P_{v,q}(f_\alpha \boxplus g_\alpha) \to 0$.

Then

$P_{v,q}(C_\varphi f_\alpha \boxplus M_{\pi_t} g_\alpha) = P_{v,q}(C_\varphi f_\alpha) \boxplus P_{v,q} M_{\pi_t} g_\alpha)$ for all $t \in \mathbb{R}$.

$$= P_{v,q}(f_\alpha \circ \varphi(x)) \boxplus P_{v,q}(\pi_t(x) g_\alpha(x))$$

$$= (\int_X (v(x) q(f_\alpha(\varphi(x))))^p d\mu)^{\frac{1}{p}} \boxplus (\int_X (v(x) q(e^{th(x)} g_\alpha(x)))^p d\mu)^{\frac{1}{p}}$$

$$= (\int_X (u(x) q(f_\alpha(x)))^p d\mu)^{\frac{1}{p}} \boxplus (\int_X \left(u(x) q(g_\alpha(x))\right)^p d\mu)^{\frac{1}{p}}$$

as $t \to 0$

$$= P_{u,q}(f_\alpha) \boxplus P_{u,q}(g_\alpha)$$

$$= P_{u,q}(f_\alpha \boxplus g_\alpha) \to 0$$

This proves that $(C_\varphi \boxplus M_{\pi_t})$ is continuous origin and hence a continuous linear operator on $MV_0(X,E) \boxplus MV_0(X,E)$.

Remark:1.2.3. Note that if $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a bounded measurable complex valued (or vector-valued) functions on X then clearly $(C_\varphi \boxplus M_{\pi_t})(f_\alpha \boxplus g_\alpha)$ is a tensor sum operator on

$MV_0(X) \boxplus MV_0(X)$ ($or \; MV_0(X,E) \boxplus MV_0(X,E)$ for any system of weights V.

If X is a system of weights generated by the characteristic functions of compact sets, then it turns out that every continuous map induces a tensor sum operators on $MV_0(X) \boxplus MV_0(X)$ ($or \; MV_0(X,E) \boxplus MV_0(X,E)$ for any system of weights V.

Theorem:1.2.4. Let X be a completely Hausdorff space and let $V = \{\lambda_{\chi k} : \lambda > 0 \; and \; K \subset X, X$ is compact$\}$.

i) Every bounded $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ on $MV_0(X) \boxplus MV_0(X)$
ii) Every bounded $\varphi : X \to X$ and $\pi_t : X \to E$ be a lmc with jointly continuous operator induces a tensor sum operator on $C_\varphi \boxplus M_{\pi_t}$ on $MV_0(X,E) \boxplus MV_0(X,E)$.

Proof: Similar proof of theorem:1.2.2

Corollary:3.9.5. Let X have the discreate topology and $V = \{\lambda_{\chi k} : \lambda \geq 0 \; and \; K \subset X, X$ is a finite set$\}$. Then every function $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ induces a tensor sum operator $(C_\varphi \boxplus M_{\pi_t})(f \boxplus g)$ on $MV_0(X) \boxplus MV_0(X)$(or $MV_0(X,E) \boxplus MV_0(X,E)$.

1.3. Dynamical system induced by tensor sum operators on weighted locally convex space of measurable functions

In this section, let us investigate the Dynamical system induced by tensor sum operators on weighted locally convex space of measurable functions

Theorem :1.3.1. Let U and V be an arbitrary system of weights on G and let $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a continuous function. Then $C_\varphi f \boxplus M_{\pi_t} g$ is a tensor sum operator for every $t \in \mathbb{R}$ and $f \boxplus g \in MV_0(X) \boxplus MV_0(X)$ iff $V\|C_\varphi \boxplus M_{\pi_t}\| \leq U$.

Proof: To show that $C_\varphi f \boxplus M_{\pi_t} g$ is a tensor sum operator. It is enough to prove $C_\varphi f \boxplus M_{\pi_t} g$ is continuous at origin. Let $v \in V$ and B_v be a neighborhood of the origin in $MV_b(X) \boxplus MV_b(X)$. Then by the given condition, there exists $u \in U$ such that $v\|C_\varphi \boxplus M_{\pi_t}\| \leq u$. Now we claim that $(C_\varphi \boxplus M_{\pi_t})(B_u) \subseteq B_v$, where B_u is neighborhood of the origin in $MU_b(X) \boxplus MU_b(X)$ $(or\ MV_b(X,E) \boxplus MV_b(X,E))$.

Let $f \boxplus g \in B_u$. Then we have

$$\|C_\varphi f \boxplus M_{\pi_t} g\|_v = \|C_\varphi f(x)\| \boxplus \|M_{\pi_t} g(x)\|$$

$$= \left(\int_X (v(x)q(\|f \circ \varphi(x)\|))^p d\mu\right)^{\frac{1}{p}} \boxplus \left(\int_X (v(x)q(\|\pi_t(x)g(x)\|))^p d\mu\right)^{\frac{1}{p}}$$

$$= \left(\int_X (v(x)q(\|f(\varphi(x))\|))^p d\mu\right)^{\frac{1}{p}} \boxplus \left(\int_X (v(x)q(e^{|t|\|h\|_\infty}\|g(x)\|)^p d\mu\right)^{\frac{1}{p}}$$

$$\leq \left(\int_X (u(x)q(\|f(x)\|))^p d\mu\right)^{\frac{1}{p}} \boxplus \left(\int_X (u(x)q(\|g(x)\|))^p d\mu\right)^{\frac{1}{p}}$$

$$\leq \|f(x) \boxplus g(x)\|$$

$$\leq \|(f \boxplus g)(x)\| \leq 1.$$

This proves that $(C_\varphi f \boxplus M_{\pi_t} g) \in B_v$ and hence $C_\varphi \boxplus M_{\pi_t}$ is a tensor sum operator.

Corollary:1.3.2. Every bounded measurable function $\varphi: X \to X$ and $\pi_t : X \to \mathbb{C}$ induces a tensor sum operator $C_\varphi \boxplus M_{\pi_t}$ on $MV_b(X) \boxplus MV_b(X)$ $(or\ MV_b(X,E) \boxplus MV_b(X,E))$ for a system of weights V on X.

Proof:

$C_\varphi\ f \boxplus M_{\pi_t} g$ is bounded, there exists $m > 0$ such that $|(C_\varphi f \boxplus M_{\pi_t} g)(x)| \leq m$ for all $x \in X$. Let $v \in X$. Then we have $v(x)|C_\varphi f \boxplus M_{\pi_t} g| \leq mv(x)$ for all $x \in X$.

Hence by the above theorem $C_\varphi f \boxplus M_{\pi_t} g$ is a tensor sum operator on $MV_b(X) \boxplus MV_b(X)$ $(or\ MV_b(X,E) \boxplus MV_b(X,E))$.

Note:1.3.3. Let $h \in F_b(\mathbb{R})$. Define $\pi_t : \mathbb{R} \to B(T)$ as $\pi_t(w) = e^{th(w)}$ for all $t, w \in \mathbb{R}$.

Theorem:1.3.4. Let $h \in F_b(\mathbb{R})$. For each $t \in \mathbb{R}$ and let $\nabla_h: \mathbb{R} \times MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T) \to M(\mathbb{R},T) \boxplus M(\mathbb{R},T)$ be the function defined by $\nabla_h(t, f \boxplus g) = C_{\varphi_t} \boxplus M_{\pi_t}(f \boxplus g)$ for all $t \in \mathbb{R}$ and $f \boxplus g \in MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T)$. Then ∇_h is a linear dynamical system on $MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T)$.

Proof:

Since $C_{\varphi_t} \boxplus M_{\pi_t}$ is a tensor sum operator on $MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T)$ for all $t \in \mathbb{R}$ and $f \boxplus g \in MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T)$. We can conclude that $\nabla_h(t, f \boxplus g) \in MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T)$. Whenever $t \in \mathbb{R}$ and $f \boxplus g \in MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T)$. Thus ∇_h is a function from $\mathbb{R} \times MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T) \to M(\mathbb{R},T) \boxplus M(\mathbb{R},T)$. It can be easily seen that $\nabla_h(0, f \boxplus g) = f \boxplus g$ and $\nabla_h(t + s, f \boxplus g) = \nabla_h(t, \nabla_h(s, f \boxplus g)$.

In order to show that ∇_h is a dynamical system on $MV_b(\mathbb{R},T) \boxplus MV_b(\mathbb{R},T)$. It is enough to show that ∇_h separately continuous map.

Let us first prove the continuity on ∇_h in the first argument. Let $\{t_n \to t\}$. Then $|t_n - t| \to 0$ as $n \to \infty$, we shall show that

$\nabla_h(t_n, f \boxplus g) \to \nabla_h(t, f \boxplus g) \in MV_b(\mathbb{R}, T) \boxplus MV_b(\mathbb{R}, T)$.

Let $v \to V$. Then,

$$P_v(\nabla_h(t_n, f \boxplus g) - \nabla_h(t, f \boxplus g))_v$$

$$= P\left(\left(C_{\varphi_{t_n}} \boxplus M_{\pi_{t_n}}\right)(f \boxplus g) - \left(C_{\varphi_t} \boxplus M_{\pi_t}\right)(f \boxplus g)\right)_v$$

$$= \left(\int_X (v(w)q(C_{\varphi_{t_n}}(\mathrm{w})f(w) \boxplus M_{\pi_{t_n}}(w)g(w)))^p d\mu)^{\frac{1}{p}}\right) -$$

$$\left(\int_X (v(w)q(C_{\varphi_t}(\mathrm{w})f(w) \boxplus M_{\pi_t}(w)g(w)))^p d\mu)^{\frac{1}{p}}\right)$$

$$= \left(\int_X (v(w)q(f(\varphi_{t_n}(w)) \boxplus \pi_{t_n}(w)g(w)))^p d\mu)^{\frac{1}{p}}\right)$$

$$+\left(\int_X (v(w)q(f(\varphi_t(w)f(w) \boxplus \pi t(w)g(w)))^p d\mu)^{\frac{1}{p}}\right)$$

$$\leq \left(\int_X (v(w)q(f(\varphi_{t_n}(w)) - e^{t_n h_\infty} g(w)))^p d\mu)^{\frac{1}{p}}\right)$$

$$+\left(\int_X (v(w)q(f(\varphi_t(w)) - e^{t_n h_\infty} g(w)))^p d\mu)^{\frac{1}{p}}\right) \quad . \quad \to$$

$0 \; as \; |t_n - t| \to 0$.

Let $f_\alpha \boxplus g_\alpha$ be a net in $MV_b(\mathbb{R}, T) \boxplus MV_b(\mathbb{R}, T)$ such that $f_\alpha \boxplus g_\alpha \to f \boxplus g$ in $MV_b(\mathbb{R}, T) \boxplus MV_b(\mathbb{R}, T)$. Then $q(f_\alpha \boxplus g_\alpha - f \boxplus g)_v \to 0$ for all $v \in V$. We shall show that $\nabla_h(t, f_\alpha \boxplus g_\alpha) \to \nabla_h(t, f \boxplus g)$ in $MV_b(\mathbb{R}, T) \boxplus MV_b(\mathbb{R}, T)$.

$$P(\nabla_h(t, f_\alpha \boxplus g_\alpha) - \nabla_h(t, f \boxplus g))_v$$

$$= P\left((C_{\varphi_t} \boxplus M_{\pi_t})(f_\alpha \boxplus g_\alpha)(w)) - ((C_{\varphi_t} \boxplus M_{\pi_t})(f \boxplus g)(w))\right)_v$$

$$= P\left((C_{\varphi_t} f_\alpha \boxplus M_{\pi_t} g_\alpha) - (C_{\varphi_t} f \boxplus M_{\pi_t} g)\right)_v$$

$$\leq \left(\int_X (v(w)q(f_\alpha(\varphi_t(w)) - f(\varphi_t(w)))e^{|t|\|h\|_\infty} q(g_\alpha(w)))^p d\mu)^{\frac{1}{p}}\right)$$

$$+\left(\int_X (v(w)q(f(\varphi_t(w)))[e^{|t|\|h\|_\infty} g_\alpha(w) - e^{|t|\|h\|_\infty} g_\alpha(w)])^p d\mu)^{\frac{1}{p}}\right)$$

$$\to 0 \; as \; (f_\alpha \boxplus g_\alpha) - (f \boxplus g) \to 0.$$

This proves the continuity of ∇_h is a (linear) dynamical system on the weighted space $MV_b(\mathbb{R}, T) \boxplus MV_b(\mathbb{R}, T)$.

1.4. Dynamical system and weighted tensor sum operator

In this section, let us investigate the dynamical system and weighted tensor sum operator

Theorem:1.4.1. Let E be a locally convex Hausdorff space such that each convergent net in E is bounded. Let $\varphi: M(X, B(E))$ and $T \in M(X,X)$. Then $(C_\varphi \boxplus M_{\pi_t})(f \boxplus g)$ is a weighted tensor sum operator on $MV_b(X,E) \boxplus MV_b(X,E)$ iff for every $v \in V$ and $p \in cs(E)$, there exists $u \in V$ and $q \in cs(E)$ such that $v(x)P(\varphi(x)(w)) \leq u(\pi_t(x)q(x)$v for all $x \in X$ and $w \in E$.

Remark:1.4.2. Let $B(E)$ be the Banach algebra of all bunded linear operators on E. Then an operator-valued map $\pi_t : X \to B(E)$ defined by $\pi_t(x) = e^{th(x)}$ for all $t \in \mathbb{R}$ and $x \in X$, where $h \in M(X, B(E))$ and $\|h\|_\infty = sup\{\|h(x)\|: x \in X\}$. Also $\varphi_t: X \to X$ is defined by $\varphi_t(x) = t + x$ the self-map. Then the weighted tensor sum operator induced by φ_t and π_t on the spaces of $MV_0(X,E)$ and $MV_0(X,E)$.

Theorem:1.4.3. Let V be an arbitrary system of weights on X. Let $\nabla: \mathbb{R} \times MV_b(X,E) \boxplus MV_b(X,E) \to M(X,E) \boxplus M(X,E)$ be the function defined by $\nabla(t, f \boxplus g) = (C_\varphi \boxplus M_{\pi_t})(f \boxplus g)$ for all $t \in \mathbb{R}$ and $f \boxplus g \in MV_b(X,E) \boxplus MV_b(X,E)$. Then ∇ is a linear dynamical system if for every $v \in V$ and $p \in cs(E)$, there exists $u \in V$ and $q \in cs(E)$ such that $v(x)P(\varphi \boxplus \pi_t)(x) \leq u(\pi_t(x)q(x)v$ for all $x \in X$ and $w \in E$.

Proof:

For every $t \in \mathbb{R}$ and $C_\varphi \boxplus M_{\pi_t}$ is a weighted tensor sum operator on $MV_b(X,E) \boxplus MV_b(X,E)$. Thus it follows that,

$\nabla(t, f \boxplus g) \in MV_b(X,E) \boxplus MV_b(X,E)$ for all $t \in \mathbb{R}$ and $f \boxplus g \in MV_b(X,E) \boxplus MV_b(X,E)$.

Clearly, ∇ is linear and $\nabla(0, f \boxplus g)(x) = (C_\varphi \boxplus M_{\pi_t})((f \boxplus g)(x))$ for all $x \in X$.

$$= (C_\varphi f \boxplus M_{\pi_t} g)(x)$$

$$= f(\varphi(x)) \boxplus e^{th(x)} g(x)$$

$$= (f \boxplus g)(x)$$

Therefore $\nabla(0, f \boxplus g)(x) = f \boxplus g$.

Also $\nabla(t+s, f \boxplus g) = \nabla(t, \nabla(s, f \boxplus g))$.

Next, to show that ∇ is a linear dynamical system, it is sufficient to show that ∇ is jointly continuous map[1]. Let $t_n \to t$ for all$t \in \mathbb{R}$. Then $t_n - t \to 0$ as $n \to \infty$. The remaining proof is similar to theorem:1.5.4

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