

VERTEX POLYNOMIAL FOR SWITCHING IN VARIOUS GRAPHS

P.Maya¹, Sanma G.R²

¹Assistant Professors, Department of Mathematics, Sree Devi Kumari Women's College, Kuzhithurai-629163, India (Affiliated to M. S. University, Tirunelveli)

² Assistant Professors, Department of Mathematics, Sree Narayana College, Varkala-695145, India
(Affiliated to University of Kerala)

Corresponding mail: drmaya009@gmail.com, sanmagr@gmail.com

Abstract

The vertex polynomial of the graph G is defined as $V(G, x) = \sum_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G) = \max \{d(v) / v \in V\}$ and v_k is the number of vertices of degree k . In this paper we found some results on vertex polynomials of switched graphs.

2010 AMS Classification : 05C12

Keywords. Vertex Polynomial, vertex switching of graphs.

1. INTRODUCTION

In this paper we mean only simple graphs. For notation and terminology, we refer to [2]. Sukumaran and Devaraj [1] introduced the concept of Vertex polynomial in graphs. Denoted vertex set by $V(G)$ and edge set by $E(G)$. For $v \in V$, $\deg(v)$ is the number of edges incident with v , the maximum degree of G is defined as $\Delta(G) = \max \{ \deg(v) / v \in V \}$. In this paper we find the vertex polynomial of switching of some graphs. The graph $G = (V, E)$ is simply denoted by G . The concept of switching was introduced by Seidel [4]. We refer [3] for more results in vertex switching. A *vertex switching* G_v of a graph is the graph obtained by taking a vertex v of G , removing the entire edges incident to v and adding edges joining v to every other vertices which are not adjacent to v in G .

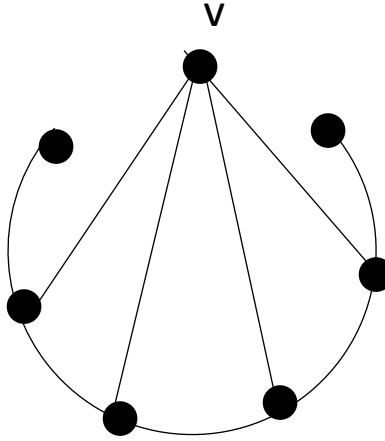


Fig.1.1

Vertex switching of v in cycle C_7 .

2. MAIN RESULTS

Theorem 2.1

The vertex polynomial for the graph obtained by switching of a vertex of a cycle C_n is $V(G, x) = x^{n-3} + (n-3)x^3 + 2x$.

Proof.

Let G denote the graph obtained by switching a vertex of cycle C_n . Let the vertex v_1 be switched.

Then the end vertices v_2 and v_n have degree 1. The vertex v_1 is adjacent to $(n-3)$ vertices and so has degree $(n-3)$. The remaining vertices $(n-3)$ has vertices will have degree 3. Hence is $V(G, x) = x^{n-3} + (n-3)x^3 + 2x$.

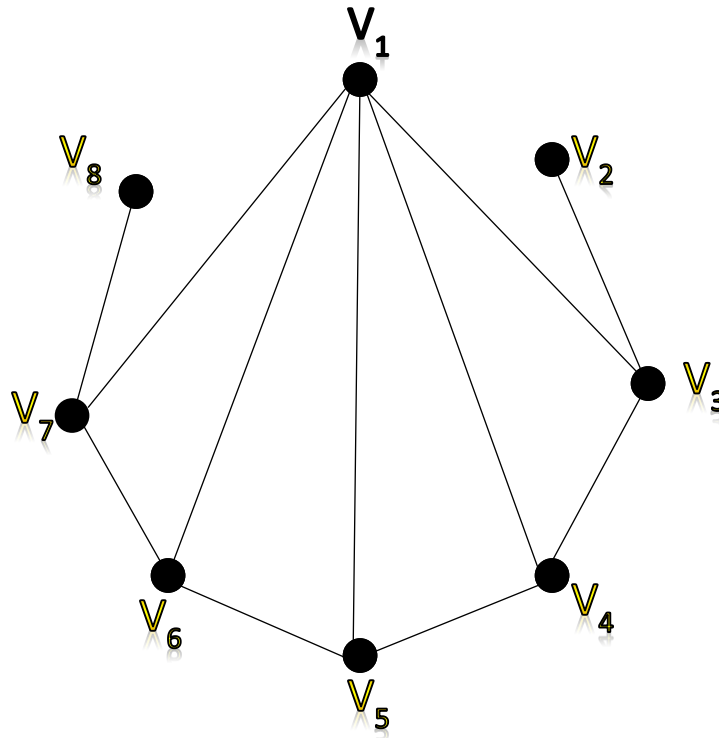


Fig.2.1

We consider the switching of Wheel graph, W_n . Here two types of switching are considered, one being switching of rim vertex and other switching of central vertex. Switching of central vertex results in isolated vertex, so we consider only the switching of a rim vertex in wheel W_n .

Theorem 2.2

The vertex polynomial for the graph obtained by switching of a rim vertex of a wheel W_n is $V(G, x) = 2x^{n-3} + (n - 3)x^4 + 2x$.

Proof.

Let G denote the graph obtained by switching of a rim vertex in the wheel W_n . Without loss of generality let us assume that the vertex v_1 is switched is shown in Fig. 2.2.

Since the vertex v_1 is switched, it is adjacent to $(n - 3)$ vertices, and so it has degree $(n - 3)$. Now the end vertices has degree 1 and the remaining vertices $(n - 3)$ vertices has degree 4.

Considering all these, the vertex polynomial is $V(G, x) = 2x^{n-3} + (n - 3)x^4 + 2x$.

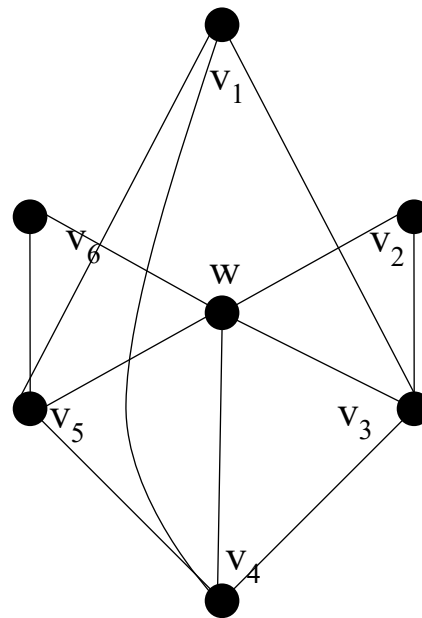


Fig.2.2

Theorem 2.3

The vertex polynomial obtained by switching a vertex of a Gear graph G_n is

$$V(G, x) = \begin{cases} x^n + nx^3 + nx^2, & \text{if central vertex is switched.} \\ x^{2(n-1)} + x^{n+1} + (n-2)x^4 + (n-1)x^3 + 2x^2, & \text{if vertex of degree 2 is switched.} \\ x^{2n-3} + x^{n-1} + (n-1)x^4 + (n-2)x^3 + 2x, & \text{if vertex of degree 3 is switched} \end{cases}$$

Proof.

CASE 1.

Let us assume that the central vertex is switched.

When the central vertex is switched it results in a wheel graph. So the vertex polynomial is $V(G, x) = x^n + nx^3 + nx^2$.

CASE 2.

Let us assume that the vertex of degree 2 is switched.

Without loss of generality let us assume that the vertex u_1 is switched. When the vertex u_1 is switched it is adjacent to $2(n - 1)$ vertices and hence having degree $2(n - 1)$. The central vertex will receive degree $(n + 1)$. The two

end vertices will have degree 2. The remaining $(n - 2)$ and $(n - 1)$ vertices will have degree 4 and 3 respectively. Hence the obtained vertex polynomial here is $V(G, x) = x^{2(n-1)} + x^{n+1} + (n - 2)x^4 + (n - 1)x^3 + 2x^2$.

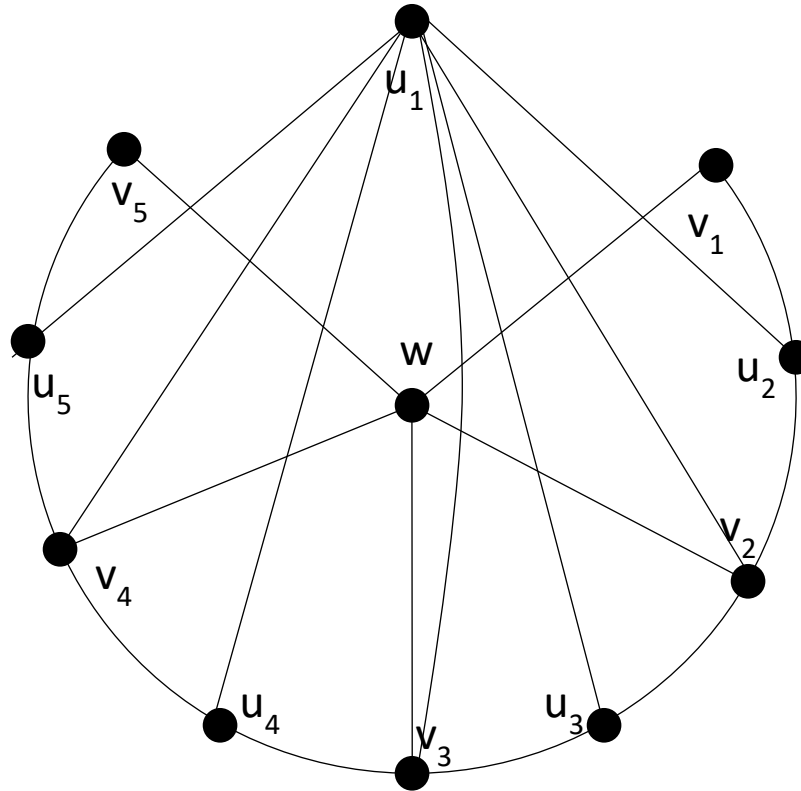


Fig.2.3

CASE 3.

Let us assume that the vertex of degree 3 is switched.

Without loss of generality let us assume that the vertex v_1 is switched. When the vertex v_1 is switched it is adjacent to $(2n - 3)$ vertices and hence having degree $(2n - 3)$. The central vertex will receive degree $(n - 1)$. The two end vertices will have degree 1. The remaining $(n - 1)$ and $(n - 2)$ vertices will have degree 4 and 3 respectively. Hence the obtained vertex polynomial here is $V(G, x) = x^{2n-3} + x^{n-1} + (n - 1)x^4 + (n - 2)x^3 + 2x$.

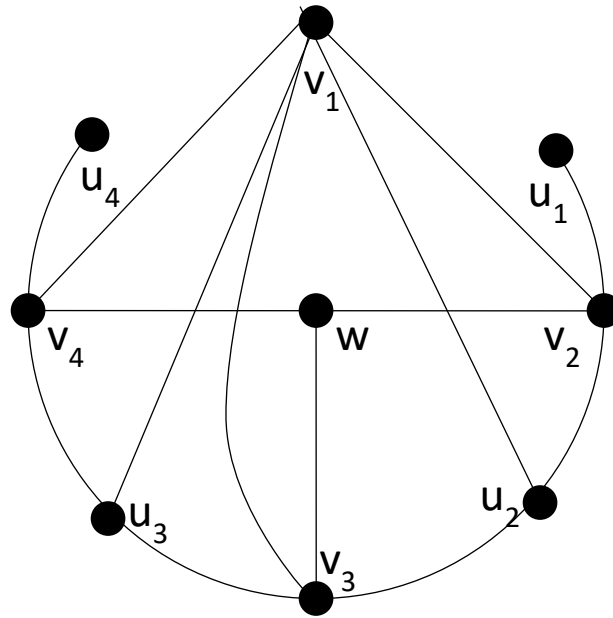


Fig.2.4

In the following theorem we consider switching of Helm graph H_n . Here we can consider only two cases, for the first case we switch the apex vertex, in the second case we switch the outer rim vertex. When we switch the inner rim vertex, the graph becomes *disconnected*.

Theorem 2.4

The vertex polynomial obtained by switching a vertex of a Helm graph H_n is

$$V(G, x) = \begin{cases} x^n + nx^3 + nx^2, & \text{if central vertex is switched} \\ x^{2n-2} + x^n + (n-1)x^5 + 9n-1)x + x^3, & \text{a outer rim vertex } u_i \text{ is switched.} \end{cases}$$

Proof.

CASE 1.

Let us assume that the central vertex is switched.

When the central vertex of a helm graph is switched, it will be adjacent to n vertices, so that it receives degree n .

The n inner vertices has degree 3 and the n outer vertices has degree 2.

Hence the vertex polynomial is $V(G, x) = x^n + nx^3 + nx^2$.

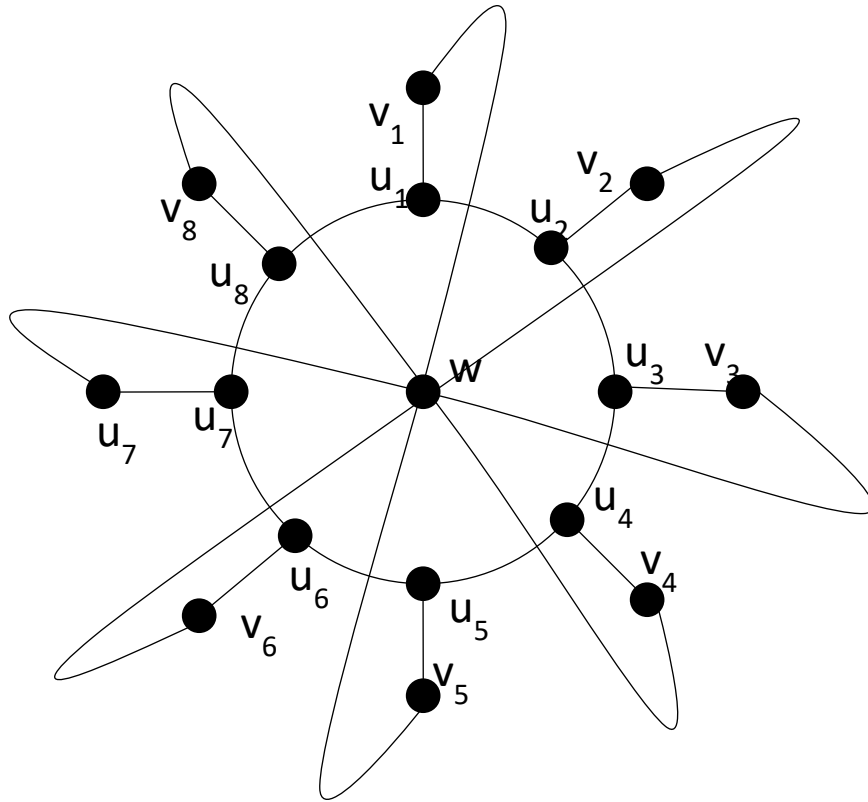


Fig.2.5

CASE 2.

Switching a outer rim vertex $u_i, 1 \leq i \leq n$

Without loss of generality let us assume that the vertex u_1 is switched.

Let the switched vertex be u_1 . Then it will be adjacent to $(2n - 2)$ vertices and so it has degree $(2n - 2)$. The central vertex is adjacent to n vertices. A inner rim vertex has degree 3 and the remaining $(n - 1)$ inner rim vertices has degree 5. The $(n - 1)$ outer rim vertices has degree 2. Hence the vertex polynomial is $V(G, x) = x^{2n-2} + x^n + (n - 1)x^5 + 9n - 1)x + x^3$.

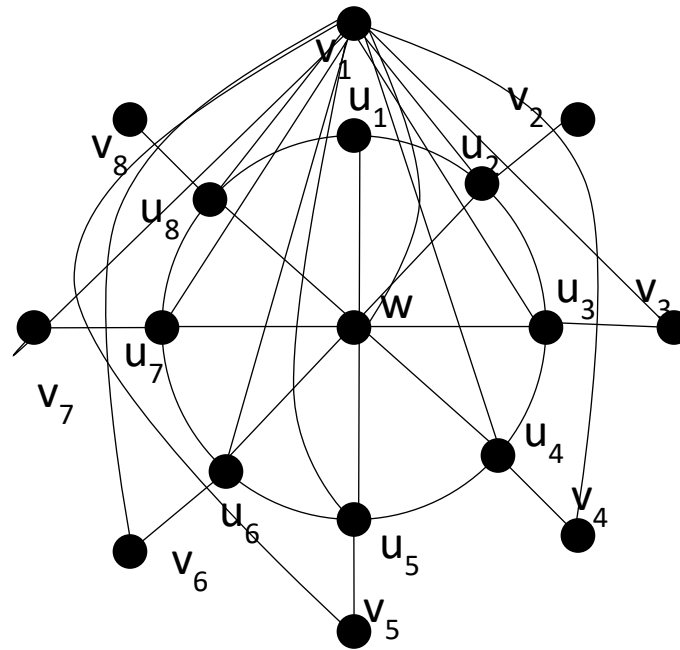


Fig.2.6

CONCLUSION

The vertex polynomial of few simple switched graph are found. This can be applied to any switched graph. These vertex polynomial of switched graph has wide applications in, circuit design, Traffic flow, Social network analysis, Computer network security, optimization, Graph Dynamics etc

References

- [1]. Devaraj J, Sukumaran E, On Vertex Polynomial, IJMSEA, 6(1)(2012), 371-380.
- [2]. Harary F, Graph Theory, Narosa Publishing House(1969).
- [3]. Maya P and Nicholas T, Duplication and switching of divisor cordial graphs, American Review of Mathematics and Statistics, ISSN: 2374 – 2348(Print), 2374 – 2356(Online), Vol. 4, No. 2, pp. 18 – 29, December 2016.
- [4]. Seidel, JJ 1976, A survey of two graphs, Proceedings of the Inter National Coll. Theorie Comb. (Rome 1973). Tomo I, Acca, Naz.Lincei, pp. 481 – 511.
- [5]. An Intercontinental Study of Employee and Employer Human Factor Issues Put Up in Aerospace and Aviation Industry - Jyothi NT, Hussainar A, Shilpa Rana, Muruga lal Jeyan JV - IJFMR Volume 6, Issue 1, January-February 2024. DOI 10.36948/ijfmr.2024.v06i01.12441
- [6]. A. S. Kumar, J. V. M. L. Jeyan, J. N. T, S. Annamalai and N. V. Kousik, "Lossless Video Compression Using Reinforcement Learning in UAV Applications," 2023 International Conference on Data Science and Network Security (ICDSNS), Tiptur, India, 2023, pp. 1-6, doi: 10.1109/ICDSNS58469.2023.10245784. 8. John B, A., Jeyan, J. V. M. L., NT, J., Kumar, A., Assessment of the Properties of Modified Pearl Millet Starch. Starch. 2022, 2200160. <https://doi.org/10.1002/star.202200160>
- [7]. Suman Rana, Bhavin Soni, Dr. P. Ebby Darney, Jyothi NT, "EFFECTS OF T4 HORMONES ON HUMAN BODY AND THEIR ANALYSIS", International Journal of Creative Research Thoughts (IJCRT), ISSN:2320-2882, Volume.10, Issue 10, pp.d332-d339, October 2022, Available at :<http://www.ijcrt.org/papers/IJCRT2210389.pdf>

- [8]. Ashika Parveen¹, JV Muruga Lal Jeyan², Jyothi NT³ International Study on Application of Value Stream Mapping to Identify the Necessity of Lean System Implementation , International Journal of Scientific Research in Engineering and Management (IJSREM) Volume: 06 Issue: 09 | September - 2022 Impact Factor: 7.185 ISSN: 2582-3930
- [9]. JV Muruga lal Jeyan, Jyothi NT Rashi Kaushik Systematic Review and Survey on Dominant Influence of Vedas and Ignorance Transpired in Space Science and Aviation", International Journal of Emerging Technologies and Innovative Research (www.jetir.org), ISSN:2349-5162, Vol.9, Issue 7, page no.b490-b493, July-2022, Available :<http://www.jetir.org/papers/JETIR2207158.pdf>
- [10]. JV Muruga lal Jeyan, Jyothi NT , Boopesh Raja, Rajarajan G "THEORY STRATEGY OF SUBSONIC WIND TUNNEL FOR LOW VELOCITY ", International Journal of Emerging Technologies and Innovative Research (www.jetir.org), ISSN:2349-5162, Vol.9, Issue 6, page no.j572-j580, June-2022, Available :<http://www.jetir.org/papers/JETIR2206973.pdf>
- [11]. JV Muruga lal Jeyan, Jyothi NT, Reshmitha Shree, Bhawadharanee S, Rajarajan, THEORETICAL STUDY OF HYPERSONIC WIND TUNNEL TEST FACILITY IN INDIA ", International Journal of Emerging Technologies and Innovative Research (www.jetir.org), ISSN:2349-5162, Vol.9, Issue 6, page no.j512-j518, June-2022, Available :<http://www.jetir.org/papers/JETIR2206967.pdf>
- [12]. JV Muruga lal Jeyan, Jyothi NT , V S Devika Thampuratty, B Nithin, Rajarajan, CONCEPT DESIGN AND DEVELOPMENT OF SUPERSONIC WIND TUNNEL ", International Journal of Emerging Technologies and Innovative Research (www.jetir.org | UGC and issn Approved), ISSN:2349-5162, Vol.9, Issue 6, page no. ppj209-j217, June-2022, Available at : <http://www.jetir.org/papers/JETIR2206925.pdf>
- [13]. Muthu Venkatesh, Rajarajan G Jyothi NT JV Muruga Lal Jeyan "Systematic Survey of Wind Tunnel Test facility in India", International Journal of Emerging Technologies and Innovative Research (www.jetir.org), ISSN:2349-5162, Vol.9, Issue 6, page no.h830-h840, June-2022, Available :<http://www.jetir.org/papers/JETIR2206795.pdf>
- [14]. Ashika Parveen, JV Muruga Lal Jeyan, Jyothi NT "Investigation Of Lean Developments And The Study Of Lean Techniques Through Event Studies" Internation Journal for Science and Advance Research In Technology, 8(4)
- [15]. P Gopala Krishnan, JV Muruga Lal Jeyan, Jyothi NT "Novel Evaluation Of Aircraft Data Structure Optimization Techniques And Opportunities" International Journal for Science and Advance Research In Technology, 8(4)
- [16]. Suryansh Upadhyay, JV Muruga lal Jeyan, Jyothi NT Preliminary Study on Brain Computer Interface © August 2021| IJIRT | Volume 8 Issue 3 | ISSN: 2349-6002 IJIRT 152537 INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN TECHNOLOGY 720
- [17]. Sruthi.s.kumar, Jyothi Nt , Jv Muruga lal jeyan . Computational Turbine Blade Analysis with Thermal Barrier Coating International Journal of Engineering Research and Applications www.ijera.com ISSN: 2248-9622, Vol. 12, Issue 4, (Series-I) April 2022, pp. 01-08, DOI: 10.9790/9622-1204010108
- [18]. A. John B, J. V. M. L. Jeyan, J. NT, A. Kumar, Assessment of the Properties of Modified Pearl Millet Starch. Starch. 2023, 75, 2200160. <https://doi.org/10.1002/star.202200160>
- [19]. John B, A., Jeyan, J. V., NT, J., & Kumar, A. (2023). Assessment of the Properties of Modified Pearl Millet Starch. Starch/Staerke, 75.
- [20]. Jyothi, N. T., Ganesan, H., & Jeyan, J. V. (2024, April). Methodical assessment and truth flow analysis of wind tunnels. In AIP Conference Proceedings (Vol. 3037, No. 1). AIP Publishing.
- [21]. Shukla, S., & Darney, P. E. The Effect of the Interfacial Resistance of the Superconducting-Stabilizer Film on the Typical Sector Diffusion Pace For 2g Hts Tapes.
- [22]. Sumalatha, M. S., & Darney, P. E. (2023). The investigation of network security, including penetration attacks and potential security mechanisms.
- [23]. lal Jeyan, J. M., Jyothi, N. T., Raja, B., & Rajarajan, G. THEORY STRATEGY OF SUBSONIC WIND TUNNEL FOR LOW VELOCITY. International Journal of Emerging Technologies and Innovative Research (www. jetir. org), ISSN, 2349-5162.

- [24]. Venkatesh, M. Rajarajan G Jyothi NT JV Muruga Lal Jeyan" Systematic Survey of Wind Tunnel Test facility in India. International Journal of Emerging Technologies and Innovative Research (www. jetir. org), ISSN, 2349-5162.
- [25]. Lal Jeyan, J. M., Jyothi, N. T., Thampuratty, V. D., Nithin, B., & Rajarajan, C. D. DEVELOPMENT OF SUPERSONIC WIND TUNNEL. International Journal of Emerging Technologies and Innovative Research (www. jetir. org| UGC and issn Approved), ISSN, 2349-5162.
- [26]. A. S. Kumar, J. V. M. L. Jeyan, J. N. T, S. Annamalai and N. V. Kousik, "Lossless Video Compression Using Reinforcement Learning in UAV Applications," 2023 International Conference on Data Science and Network Security (ICDSNS), Tiptur, India, 2023, pp. 1-6, doi: 10.1109/ICDSNS58469.2023.10245784.
keywords: {Image coding;Neural networks;Data compression;Reinforcement learning;Video compression;Network security;Data science;Lossless Video;Compression;Reinforcement Learning;UAV},